Question Paper

Physics Numerical Type (Maximum Marks: 21)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

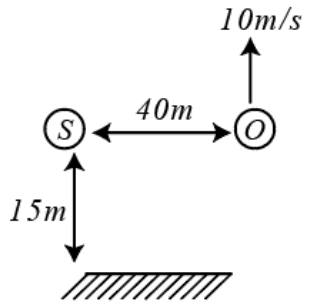
2 moles of a diatomic gas undergoes a thermodynamic process $\frac{PT^2}{V}$ = constant. The molar heat capacity of the gas is $\frac{nR}{2}$. Find n.

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

S is a stationary source of sound emitting waves of frequency 440 Hz. These waves are reflected by a stationary wall and heard by observer O moving away with v = 10 m/s . Find beat frequency (in Hz) heard by observer in the position shown (due to superposition between reflected & direct waves) (V_{sound} = 330 m/s)

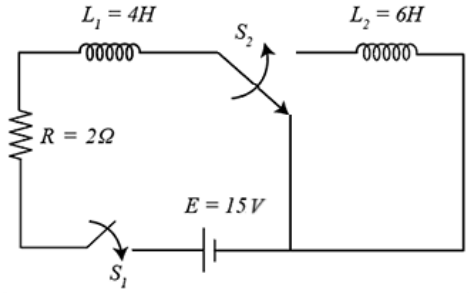


Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

In the given linear electrical circuit, two operations are performed:
Operation 1: S_1 has been closed at time t = 0
Operation 2: S_2 has been toggled at time t = 2 s
Then, the value of the Current passing through the Resistor immediately after Operation 2 has been performed is _____ mA. (It is given that the Value of 1/e = 0.37)



Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

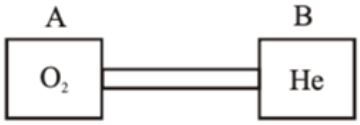
Radiation from hydrogen gas excited to first excited state is used for illuminating certain photoelectric plate. When the radiation from some unknown hydrogen like gas excited to the same level is used to expose the same plate, it is found that the de-Broglie wavelength of the fastest photoelectron has decreased 2.3 times. It is given that the energy corresponding to the longest wavelength of the Lyman series of the unknown gas is 3 times the ionization energy of hydrogen gas (13.6eV) . Find the work function of photoelectric plate in eV. (Take $(2.3)^2 = 5.25$) .

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

Two containers A and B are connected by a conducting solid cylindrical rod of length $\frac{242}{7}$ cm and radius $\sqrt{8.3}$ cm. Thermal conductivity of the rod is 693 watt/mole-K. The container A contains two mole of oxygen gas and the container B contains four mole of helium gas. At time $t = 0$ temperature difference of the containers is 50°C, after what time (in seconds) temperature difference between them will be 25°C. Transfer of heat takes place through the rod only. Neglect radiation loss. Take R = 8.3 J/ mole −K , $ln2 = 0.693$ and $\pi = \frac{22}{7}$.

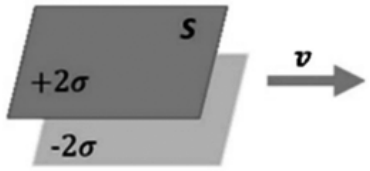


Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

As shown in the figure below, consider a large parallel plate capacitor with uniform surface charges 2σ and -2σ on the upper and the lower plates, respectively. These charges are moving with a constant speed V. The area of each plate is given as S. Find the value of K such that magnetic field between the plates is B = K($\mu\sigma$V) .

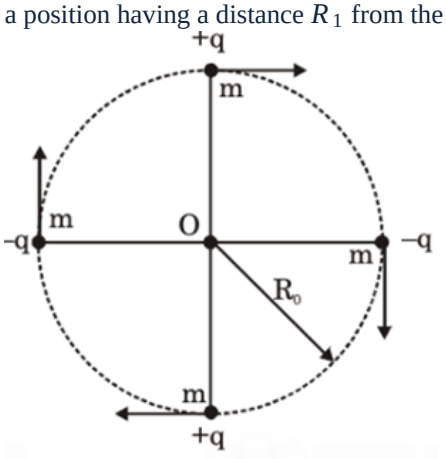


Question No. 7

Numerical Type

The answer has to be filled into the input box provided below.

In free space on a circle with radius R_0, four point masses m are located at the vertices of the inscribed square, two of them carry charge $+q$, and the other two charge $-q$. At the initial moment all these particles have same speed in clockwise direction as shown. It is known that during the motion the minimum distance from any of the point masses to the centre O of the initial circle is $R_1(R_1 < R_0)$. Consider that at any time, the charges are at the vertices of square centered at point O. The action of gravitational forces can be neglected. Determine the speed (in m/s) of any particle at a position having a distance R_1 from the centre of the circle. Take $m = 2\sqrt{2} - 1$gm , q = 1μC, $R_0 = 2R_1 = 1$ m .



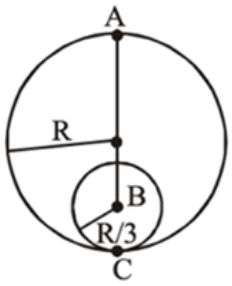
Physics Multiple Correct (Maximum Marks: 28)

Question No. 1

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Inside a uniform sphere of mass M and radius R, a cavity of radius $R/3$ is made in the sphere as shown.



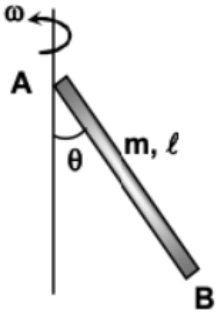
A. Gravitational field inside the cavity is uniform.
B. Gravitational field inside the cavity is non-uniform.
C. The escape velocity of a particle projected from point A is $\sqrt{\frac{88GM}{45R}}$.
D. Escape velocity is defined for earth and particle system only.

Question No. 2

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

A thin uniform rod AB of mass $m = 1$ kg and length $\ell = 1$ m is hinged at end 'A'. The rod makes an angle $\theta = 53^\circ$ with the vertical axis and it is rotating with a constant angular velocity 'ω' about the vertical axis passing through end 'A' as shown in the figure. Then choose the correct option(s).



A. The angular velocity 'ω' must be 5rad/s .
B. The angular velocity 'ω' must be 10rad/s .
C. The net hinge reaction on the rod at end 'A' is $10\sqrt{2}$ N.
D. The net hinge reaction on the rod at end 'A' is 10 N.

Question No. 3

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Which of the following statements with regards to the Single Slit Fraunhoffer Diffraction and Polarisation of Light is(are) true?

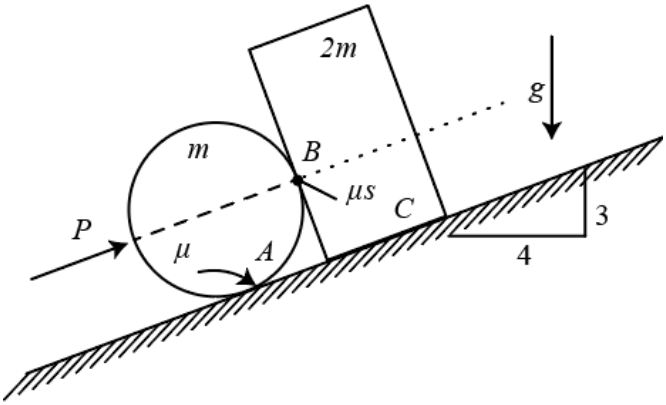
A. All the Diffraction Maximas are equidistant from each other.
B. With regards to Polarisation, the Difference between the transmitted light Intensity and original light intensity is proportional to the square of the polarisation angle, for very small values of the Polarization angle. (Assume that Transmission has occurred only once)
C. The Graph of the Intensity vs angle of diffraction is not Monotonically Decreasing for Positive values of the angle of Diffraction.
D. For very small Brewster's angle, the Relative refractive index between two media is proportional to the Brewster's angle.

Question No. 4

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Consider the system of a rough cylinder and a block on an inclined plane with an external force P along the incline in an impending motion up the incline. Static coefficient between cylinder and plane as well as the block and the plane is μ. Assume impending slip at A and C. What is the limiting value of the static coefficient μ_s at B such that the impending motion is of the given case and what is the corresponding value of P.



A. $\mu_s = \frac{2\mu}{6\mu^2+7\mu+3}$
B. $P = \mu mg\cos\theta + mg\sin\theta$
C. $\mu_s = \frac{4\mu}{6\mu^2+7\mu+3}$
D. $P = 3\mu mg\cos\theta + mg\sin\theta$

Question No. 5

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

A bowl of water is placed on one of the plates of a beam scale, which is balanced. One of our fingers is immersed into the water, such that the bowl is not touched. Will the scale remain balanced or not, if (i) not a drop of water flows out of the bowl; (ii) the bowl was full to the brim, and the water flows off the plate of the scale?

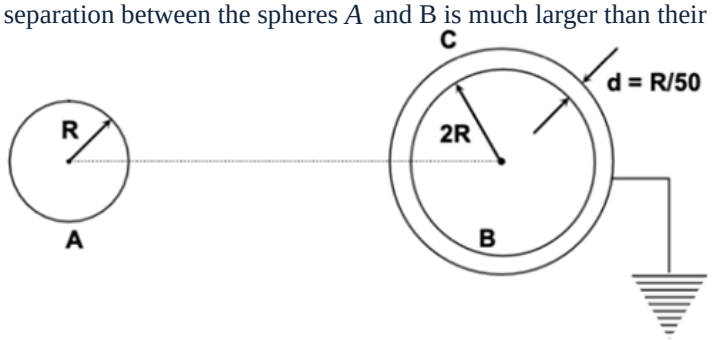
A. In case (i) the scale remains balanced.
B. In case (i) the bowl of water weighs more.
C. In case (ii) the bowl of water weighs less.
D. In case (ii) the bowl of water weighs the same.

Question No. 6

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Two conducting spheres A and B of radius R and $2R$ are connected by a thin conducting wire. The conducting sphere ' B ' is surrounded by a grounded thin concentric conducting shell ' C ' with radius $(\frac{101}{50}R)$. The connecting wire between the spheres A and B is not touching the thin shell ' C '. If conducting sphere ' B ' is given a charge ' Q ' and the separation between the spheres A and B is much larger than their radii. Then choose the correct option(s).



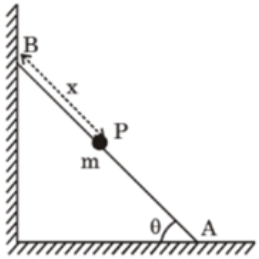
A. The charge appearing on the conducting sphere ' A ' is $\frac{Q}{201}$.
B. The charge appearing on the conducting sphere ' A ' is $\frac{Q}{101}$.
C. The charge appearing on the conducting sphere ' B ' is $\frac{100Q}{101}$.
D. The charge appearing on the conducting sphere ' B ' is $\frac{200Q}{201}$.

Question No. 7

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

A light stick of length ℓ rests with its one end against the smooth wall and other end against the smooth horizontal floor as shown in the figure. The bug starts at rest from point B and moves such that the stick always remains at rest, a_p is the magnitude of acceleration of bug of mass m, which depends upon its distance of x from the top end of the stick. Choose the CORRECT option(s) :



A. $a_p = \frac{g}{\sin\theta}\left(1 - \frac{x}{\ell}\right)$

B. $a_p = \frac{g}{\cos\theta}\left(1 - \frac{x}{\ell}\right)$

C. The time taken by the bug to reach the bottom of the stick having started at the top end from rest is $\frac{\pi}{2}\sqrt{\frac{\ell\sin\theta}{3g}}$.

D. The time taken by the bug to reach the bottom of the stick having started at the top end from rest is $\frac{\pi}{2}\sqrt{\frac{\ell\sin\theta}{g}}$.

Physics Question Stem (Maximum Marks: 12)

Question No. 1

Numerical Type

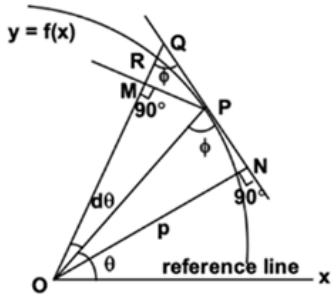
The answer has to be filled into the input box provided below.

Question Stem

If a point P moves in plane along a given curve $y = f(x)$, the angular velocity of point P about a fixed point O in the plane is the rate of change of the angle that OP line makes with a fixed direction [$OX -$ line] in the plane

Let $OP = r$ at $t =$ t sec

$PM = rd\theta = PQ\sin\phi$, But if $d\theta$ is very small then. $PQ \cong PR = ds$ (arc length)



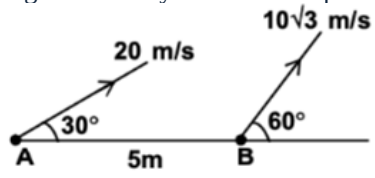
$\Rightarrow rd\theta = ds\sin\phi$

$\Rightarrow \frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt}\sin\phi = \frac{v\sin\phi}{r} = \frac{\text{Magnitude of component of velocity of point perpendicular to radius vector}}{\text{Magnitude of radius vector}}$

$\Rightarrow \frac{d\theta}{dt} = \frac{vp}{r^2}$

Question

If two particles A and B are having speed $10\sqrt{3}$ m/s and 20 m/s at a particular instant as shown in the figure, then the angular velocity of A with respect to B at the same instant is mark absolute value in rad/s.



Question No. 2

Numerical Type

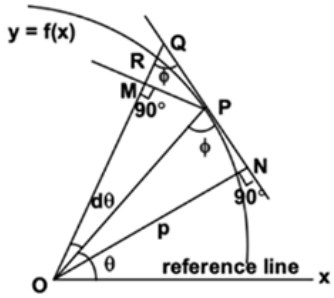
The answer has to be filled into the input box provided below.

Question Stem

If a point P moves in plane along a given curve $y = f(x)$, the angular velocity of point P about a fixed point O in the plane is the rate of change of the angle that OP line makes with a fixed direction [$OX -$ line] in the plane

Let $OP = r$ at $t =$ t sec

$PM = rd\theta = PQ\sin\phi$, But if $d\theta$ is very small then. $PQ \cong PR = ds$ (arc length)



$\Rightarrow rd\theta = ds\sin\phi$

$$\Rightarrow \frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt}\sin\phi = \frac{v\sin\phi}{r} = \frac{\text{Magnitude of component of velocity of point perpendicular to radius vector}}{\text{Magnitude of radius vector}}$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{vp}{r^2}$$

Question

If point P moves on parabolic path $y^2 = 4(x + 1)$, where x and y are in meter with constant speed 2 m/s . Its angular velocity (in rad/sec) about focus at an instant when it makes angle 60° at focus with x-axis is [all angles are measured in anticlockwise direction with positive x-axis]

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

Question Stem

If two deuterium nuclei get close enough together, the attraction of the strong nuclear force will fuse them to make as isotope of helium. The process will release a vast amount of energy. The range of nuclear force is 10^{-15} m. This is the principle behind the nuclear fusion reactor. The deuterium nuclei moves so fast hence it is not possible to contain them by physical walls. Therefore, they are confined magnetically.

Question

The strength of magnetic field (in mT) required to make deuterium nuclei moving with this speed, to be confined in a circle of diameter 2.5 m

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

Question Stem

If two deuterium nuclei get close enough together, the attraction of the strong nuclear force will fuse them to make as isotope of helium. The process will release a vast amount of energy. The range of nuclear force is 10^{-15} m. This is the principle behind the nuclear fusion reactor. The deuterium nuclei moves so fast hence it is not possible to contain them by physical walls. Therefore, they are confined magnetically.

Question

The following is the process of formation of the Helium nuclei from the nuclear fusion of deuterium nuclei (Given 1 amu = 931.5Mev/c^2)

$${}_1^2H + {}_1^2H \rightarrow {}_1^3H + {}_1^1H + \Delta E_1$$

$${}_1^3H + {}_1^2H \rightarrow {}_2^4\text{He} + {}_0^1n + \Delta E_2$$

[$m({}_1^2H)$ = 2.014102 a.m.u, $m({}_1^3H)$ = 3.016049 a.m.u, $m({}_2^4He)$ = 4.002603 a.m.u,

$m({}_0^1n)$ = 1.008665 a.m.u, $m({}_1^1H)$ = 1.007825 a.m.u] then $\Delta E_1 + \Delta E_2$ is - (in MeV round off to nearest integer).

Chemistry Numerical Type (Maximum Marks: 21)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

A solution prepared by mixing 10 mL of a 0.10M solution of the R enantiomer of a compound and 30 mL of a 0.10M solution of the S enantiomer was found to have an observed specific rotation of +4.8. What is the absolute value of specific rotation of each of the enantiomers? (Hint: mL × M = millimole, abbreviated as mmol)

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

A solution is created by mixing equal volumes of 0.1M Na_2SO_4 and 0.1M $BaCl_2$. The resultant solution volume is 1 litre, its temperature rises to $'T' = 50^\circ C$ and it also contains a precipitate mass. Through highly accurate measurements, total ionic concentration is found to be 0.20005 M (excluding H^+ and OH^- ions). Then report the number of correct statement(s) among the following

(A) Final solution contains equal moles of all ions (excluding H^+ and OH^- ions)

(B) Final solution contains equal moles of Na^+ and Cl^- ions.

(C) Final solution contains equal moles of Ba^{+2} and SO_4^{2-} ions.

(D) Ksp value of $BaSO_4$ is 2.5×10^{-9} at temperature $'T'$

(E) pH of resultant solution is 7

(F) Resultant solution after filteration can act as buffer solution

(G) Final solution is neutral because has $[H^+]_{eqm} = [OH^-]_{eqm}$

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

A quantity of 25 mL of solution containing both Fe^{2+} and Fe^{3+} ions is titrated with 25 mL of 0.0200 M $KMnO_4$ and dilute H_2SO_4. As a result all the Fe^{2+} ions are oxidised to Fe^{3+} ions. Next 25 mL of the original solution is treated with Zn metal. Finally, the solution requires 40 mL of the same $KMnO_4$ solution for oxidation to Fe^{3+}.

$$MnO_4^- + 5Fe^{2+} + 8H^+ \longrightarrow Mn^{2+} + 5Fe^{3+} + 4H_2O$$

Molar concentration of Fe^{3+} in original solution is

Question No. 4

Numerical Type

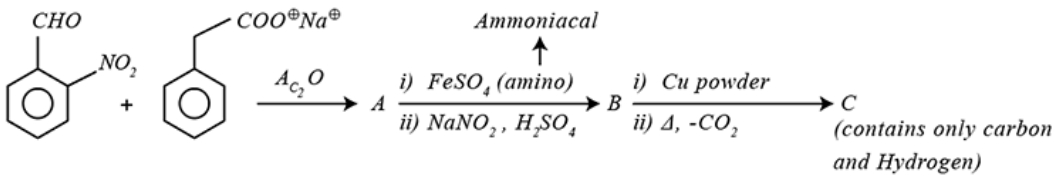
The answer has to be filled into the input box provided below.

How many of the following polymers contain atoms of halogen family in their structures?
PVC, orlon, teflon, terylene, neoprene, bakelite

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.



Degree of Unsaturation of C ?

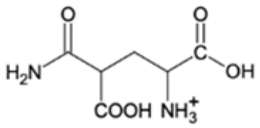
Question No. 6

Numerical Type

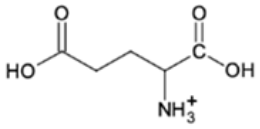
The answer has to be filled into the input box provided below.

Some amino acids are given below:

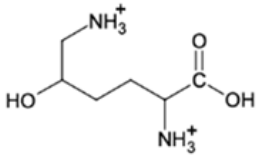
(I)



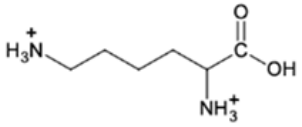
(II)



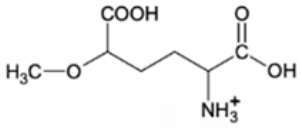
(III)



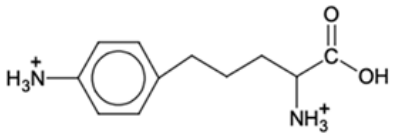
(IV)



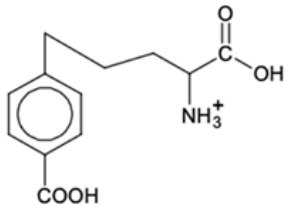
(V)



(VI)



(VII)



Let, the number of amino acids which exists as cation at $pH = 7$ be 'x' and the number of amino acids which exists as anion at $pH = 7$ be 'y', then find the value of $\frac{x}{y}$.

Question No. 7

Numerical Type

The answer has to be filled into the input box provided below.

Find the value of $\frac{n}{4}$, where n is the number of compounds will produce $POCl_3$ when treated with PCl_5 as intermediate product or final product.

$CH_3COOH, P_4O_{10}, H_2O, H_2SO_4, C_2H_5OH$

Chemistry Multiple Correct (Maximum Marks: 28)

Question No. 1

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Radionucleide 'P' decays into 'Q' and 'Q' decays into 'R' as :

$$P \xrightarrow[K_1]{t_{1/2}=1000hrs} Q \xrightarrow[K_2]{t_{1/2}=10\ min} R$$

Number of nuclei of Q and R after 1000hr, if we start with 2 mol P, are.

$[N_A = 6 \times 10^{23}]$

A. $N_Q = 6 \times 10^{23}$
B. $N_Q = 10^{20}$
C. $N_R = 6 \times 10^{23}$
D. $N_R = 10^{23}$

Question No. 2

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

The radial distribution functions $[P(r)]$ is used to determine the most probable radius, which is used to find the electron in a given orbital $\frac{dP(r)}{dr}$ for 1 s-orbital of hydrogen like atom having atomic number Z, is

$\frac{dP(r)}{dr} = \frac{16\pi Z^3}{a_0^3}(2r - \frac{2Zr^2}{a_0})e^{-2Zr/a_0}$

Which of the following statement(s) is/are correct?

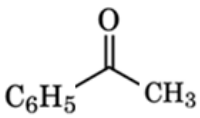
A. At the point of maximum value of radial distribution function, $\frac{dP(r)}{dr} = 0$; one antinode is present
B. Most probable radius of Li^{2+} is $\frac{a_0}{3}$
C. Most probable radius of He^+ is $\frac{a_0}{2}$
D. Most probable radius of hydrogen atom is a_0

Question No. 3

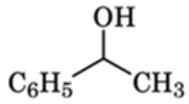
One or More Options Correct Type

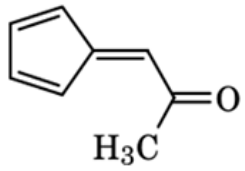
The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Compound (X), molecular formula C_8H_8O, is water insoluble and gives yellow ppt. with $I_2/NaOH$. It reacts with H_2N-OH in the presence of NH_4Cl/NH_4OH to give two geometrical isomers, which can go for rearrangement with acids. Compound (X) can be,

A. 

B. 

C. 

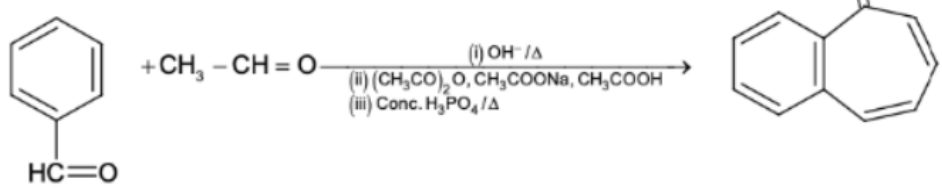
D. 

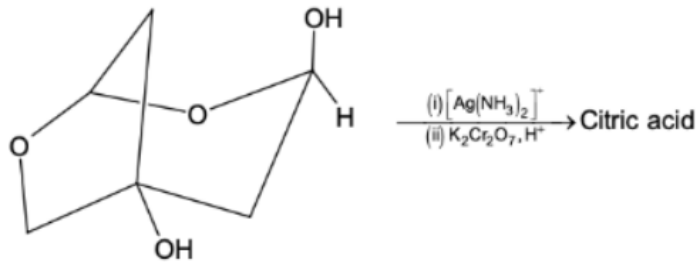
Question No. 4

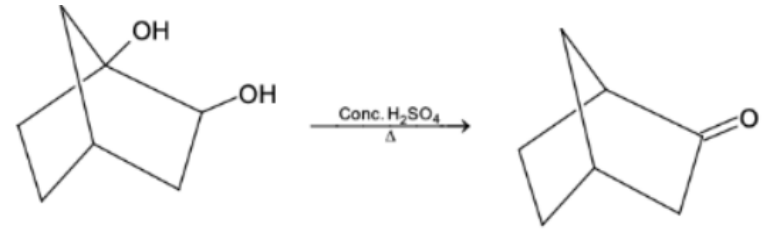
One or More Options Correct Type

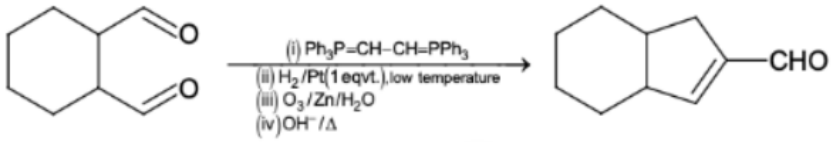
The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Identify the reaction(s) with their correct major product?

A. 

B. 

C. 

D. 

Question No. 5

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Cu^{2+} (aq.) + Zn(s) $\longrightarrow$ Cu(s) + Zn^{2+} (aq.) ; E^0 = 1.1 V

Zn^{2+} (aq.) + $2e^-$ $\longrightarrow$ Zn(s); E^0 = −0.76 V

Cu^{2+} (aq.) + $4NH_3$(aq.) $\longrightarrow$ $[Cu(NH_3)_4]^{2+}$ (aq.) $K_{eq} = 10^{20}$

If $\frac{2.303RT}{F} = 0.06$ then the E^0 of following reaction is -

Cu(s) + $4NH_3$ (aq.) $\longrightarrow$ $[Cu(NH_3)_4]^{2+}$ (aq.) + $2e^-$

A. 0.34 V
B. 0.76 V
C. 0.26 V
D. 0.14 V

Question No. 6

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

A crystalline cubic solid is made up of atoms A, B, C and D such that atoms ' A ' are present at each of the corner, ' B ' atoms at each of face centre, ' C ' atoms in each of octahedral void and ' D ' atoms in each of the tetrahedral voids of the unit cell. Now choose the correct statement(s) regarding the above solid.
(Atoms from the lattice sites of the unit cell have to removed completely and not partially)

A. If all atoms lying along or inside one of the octant are removed, then the formula of resulting solid would be $A_7B_{16}C_{12}D_{56}$.
B. If all atoms lying along one of the C_4 − axis of symmetry are removed, then the formula of the resulting solid would be $AB_2C_3D_8$.
C. If all atoms lying along one of the C_3 - axis of symmetry are removed, then the formula of resulting solid would be $AB_4C_4D_8$.
D. If all the atoms lying along one of the C_2 - axis of symmetry are removed, then the formula of resulting solid would be A_2 $B_6C_5D_{16}$.

Question No. 7

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

$$[X] + H_2O_2 + H^+ \longrightarrow \underset{\text{(Blue colour)}}{[Y]} + H_2O$$

$$[Y] + H^+ \longrightarrow \underset{\text{(Green soln)}}{[Z]} + M \uparrow + H_2O$$

Now, identify the correct statement(s) regarding above sequence of reactions:

A. ' X ' may be K_2CrO_4 or $K_2Cr_2O_7$.
B. ' Y ' is CrO_3.
C. Green colour of (Z) is due to the presence of Cr^{3+}.
D. Gas (M) is O_2.

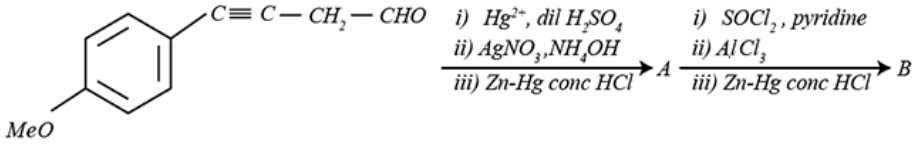
Chemistry Question Stem (Maximum Marks: 12)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

Question Stem



Question

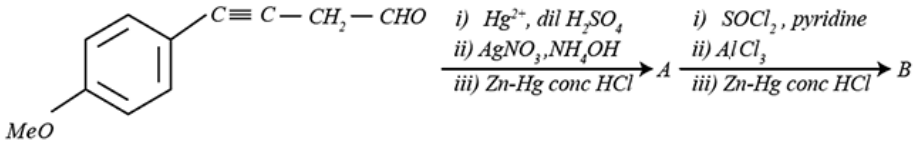
Degree of Unsaturation of A is x and number of oxygen is y in A. Find $x + y$ =

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

Question Stem



Question

Molecular mass of B is m.
Find $m/10$. _____

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

Question Stem

Addition of non-volatile solute always lowers the vapour pressure of solvent, therefore, it will be in equilibrium with solid phase at a lower pressure and hence at a lower temperature, the difference between the freezing points of the pure solvent and its solution is called depression of freezing point. The depression in freezing point when 1 mol of solute is dissolved in 1000gm of solvent is called molal freezing point depression constant (K_f). The value of K_f for H_2O is $1.86\ K\ mol^{-1}\ kg$. To understand this, a solution of 25%(w/w)$MgCl_2$ containing impurity of the chloride of a trivalent metal M having atomic mass 43.5gm(24% by weight of $MgCl_2$) has been prepared. In this solution $MgCl_2$ is completely dissociated. On the other hand only 50% dissociation of MCl_3 takes place and molality is approximately equal to molarity.

Question

Depression in freezing point of the solution is (mark answer to two decimal places)

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

Question Stem

Addition of non-volatile solute always lowers the vapour pressure of solvent, therefore, it will be in equilibrium with solid phase at a lower pressure and hence at a lower temperature, the difference between the freezing points of the pure solvent and its solution is called depression of freezing point. The depression in freezing point when 1 mol of solute is dissolved in 1000gm of solvent is called molal freezing point depression constant (K_f). The value of K_f for H_2O is 1.86 K mol^{-1} kg. To understand this, a solution of 25%(w/w)$MgCl_2$ containing impurity of the chloride of a trivalent metal M having atomic mass 43.5gm(24% by weight of $MgCl_2$) has been prepared. In this solution $MgCl_2$ is completely dissociated. On the other hand only 50% dissociation of MCl_3 takes place and molality is approximately equal to molarity.

Question

If the solution is cooled to $-21.7^{\circ}C$, what weight of ice will be formed?

Mathematics Numerical Type (Maximum Marks: 21)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

Let $(1 + x + x^2)^{20} = \sum_{k=0}^{40} a_k x^k$. If $\alpha a_{11} = \beta a_{10} + \gamma a_9$, where $\alpha, \beta, \gamma \in N$, then the value of $\frac{\gamma - \alpha}{\beta}$ is

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

Let $a_1, a_2, a_3 \ldots a_n (n \in N)$ is a sequence such that $a_1 = a_2 = 2$ and $\frac{2a_{n-1}\, a_n}{a_{n-1}\, a_{n+1} - a_n^2} = n^3 - n$, $\forall n = 2, 3, 4, \ldots$, find the value of $\frac{[S]}{16}$, where $S = \sum_{k=2}^{2020} \frac{a_{k+1}}{a_k}$ and ([.] denotes G.I.F.)

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

Consider, $f(x) = \frac{|x-4|}{|x|+1}$. If sum of all distinct possible values of $\sin^{-1}(\sin[f(x)])$ is $a\pi + b$, then find the absolute value of $(a + b)$.

[Note: $[z]$ denotes greatest integer function less than or equal to z.]

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

Let $S_n = \frac{1}{1^n} + \frac{1}{3^n} + \frac{1}{5^n} + \ldots \infty$ and $\int_0^1 \frac{x(\ln x)^2}{1-x^4}\, dx = mS_3$, then value of m is

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

Let A be a 2×2 matrix, and suppose that $A^2 = 0$. Then for each scalar c, $\det(cI - A) = c^x$. Find x.

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

Let A_1, A_2, A_3, A_4 be the areas of the triangular faces of a tetrahedron, and h_1, h_2, h_3, h_4 be the corresponding altitude of the tetrahedron. If volume of tetrahedron is $1/6$ cubic units, then find the minimum value of $(A_1 + A_2 + A_3 + A_4)(h_1 + h_2 + h_3 + h_4)$ (in cubic units).

Question No. 7

Numerical Type

The answer has to be filled into the input box provided below.

Six points $(x_i, y_i), i = 1, 2, \ldots..6$ are taken on the circle $x^2 + y^2 = 4$ such that $\sum_{i=1}^{6} x_i = 8$ and $\sum_{i=1}^{6} y_i = 4$. The line segment joining orthocenter of a triangle made by any three points and the centroid of the triangle made by other three points passes through a fixed point (h, k). Find the value of $h + k$.

Mathematics Multiple Correct (Maximum Marks: 28)

Question No. 1

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

$f(x)$ is a twice differentiable function such that
$f(x) + f''(x) = -xg(x)f'(x), \quad g(x) \geqslant 0 \quad \forall \quad x \geqslant 0$ then:

A. If $f(x)$ is an increasing function for $x \geqslant 0$, then $f''(1) \leq -f(1)$
B. $|f(x)| \leqslant \sqrt{f(0)^2 + f'(0)^2}$ for $x \geqslant 0$
C. $f'(a)(f(1) + f(0)) + f''(b)(f'(0) + f'(1)) > 0$ for some $a, b \in (0, 1)$
D. If f is increasing with upward concavity then, $\exists \alpha \in (0, \infty)$ for which $f(x) > 0$

Question No. 2

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

$P(x)$ is a least degree polynomial such that $(P(x) - 1)$ is divisible by $(x - 1)^2$ & $(P(x) - 3)$ is divisible by $(x + 1)^2$, then

A. graph of $y = P(x)$ is symmetric about origin
B. $y = P(x)$ has two points of extrema
C. $\int_{-4}^{\lambda} P(x)dx = 0$ for exactly one value of λ.
D. $\int_{-4}^{\lambda} P(x)dx = 0$ for exactly two values of λ.

Question No. 3

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Let $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}; x, y, z \in$ I with the condition that $\det(\text{adj}(\text{adj}\, A)) = 2^{12} \times 3^8 \times 5^4$, S_1 is set of all the matrices A, S_2 is set of all such matrices A where trace of A is divisible by 2, and $x, y, z \in N$ and S_3 is set of all such matrices A, where $(\text{trace}\, A^3) - 3 \det A = 0$, and $x, y, z \in N$. Let the number of elements in set S_1, S_2 and S_3 be $2\alpha, \beta$ and γ respectively.
Which of the following is/are correct?

A. $\frac{\alpha}{\beta+\gamma} < 10$
B. $\frac{\alpha}{\beta-\gamma} > 15$
C. $\frac{\alpha}{\beta+\gamma} > 9$
D. $\frac{\alpha}{\beta-\gamma} < 14$

Question No. 4

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

A differentiable function $f : R \to R$ satisfies following relations

(i) $f(x) - f(y) = (x - y)f'\left(\frac{x+y}{2}\right), \forall x, y \in R$

(ii) $f(0) = f(1) = 0, f(2) > 0$

which of the following is/are true

A. $f(x)$ is an onto function

B. $f(x)$ has an extrema at $x = \frac{1}{2}$

C. $f(x)$ is a many one function

D. $\lim_{x \to -\infty} f(x) = -\infty$

Question No. 5

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

If z is a unimodular complex number such that $\text{Re}(z - 1) + \text{Re}(z^2) = \int_0^{\pi/2} \sin x \ln|\sin x - \cos x| dx$, then z can satisfy

A. $z + \bar{z} = -2$

B. $z + \bar{z} = 1$

C. $\arg(z) = \frac{\pi}{3}$

D. $\arg z = \pi$

Question No. 6

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

Let a ray of light through $P(10, 25, 35)$ is incident at point Q on the plane $x + y + z = 100$ and passes through the point $R(40, 60, 60)$ after reflection, then

A. coordinates of foot of perpendicular from P on the given plane is $(20, 45, 35)$

B. equation of incident ray is $\frac{x-10}{2} = \frac{y-25}{1} = \frac{z-35}{3}$

C. equation of reflected ray is $\frac{x-40}{2} = \frac{y-60}{3} = \frac{z-60}{1}$

D. equation of plane containing incident ray and reflected ray is $2x - y - z + 40 = 0$

Question No. 7

One or More Options Correct Type

The question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

$P(x)$ is a fourth degree polynomial such that

(1) $P(-x) = P(x) \forall x \in R$,

(2) $P(x) > 0 \forall x \in R$,

(3) $P(0) = 1$

(4) $P(x)$ has exactly two local minima at x_1 and x_2 such that $|x_1 - x_2| = 2$.

The line $y = 1$ touches the curve at a certain point Q and the enclosed area between the line and the curve is $\frac{8\sqrt{2}}{15}$. Let $g(x) = Ax^2 + Bx + C(A \neq 0)$ such that $\lim_{x \to 0} \frac{P(x) - g(x) - g(-x)}{x^2}$ is finite and is equal to the slope of the tangent of $g(x)$ at $x = -1$. Also, $P(x)$ and $g(x)$ have common tangent as $y = 1$.

A. the value of A is $-\frac{1}{2}$

B. the value of $B + C$ is $-\frac{1}{2}$

C. the value of $A + C$ is 1

D. the value of $A + B + C$ is -1

Mathematics Question Stem (Maximum Marks: 12)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

Question Stem

It is given $I_s = \int_0^\infty e^{-t} \cdot t^{s-1}\, dt, (s > 0)$ and $I_p \cdot I_q = \frac{\pi}{\sin(p\pi)}$ (where $0 < p \le q < 1$ & $p + q = 1$) .

Question

$\prod_{i=1}^{9} I_{(\frac{i}{10})} = k(\sqrt{\frac{\pi^{\alpha}}{5}})$, where k and α are natural numbers, then value of $(\alpha + k)$ is -

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

Question Stem

It is given $I_s = \int_0^\infty e^{-t} \cdot t^{s-1}\, dt, (s > 0)$ and $I_p \cdot I_q = \frac{\pi}{\sin(p\pi)}$ (where $0 < p \le q < 1$ & $p + q = 1$) .

Question

$A = \int_0^\infty x^2 e^{-x^4}\, dx$ and $B = \int_0^\infty e^{-x^4}\, dx$. If the value of AB is $\frac{1}{k}$, then $[k]$ is

(Note : [k] is greatest integer less than or equal to k)

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

Question Stem

Using three basic colours (Red, Blue, Green) different colour are made by mixing these two or more colours in equal proportion. The n vertical stripes are painted by all colours available, such that no two consecutive stripes have same colour.

Question

For $n = 13$, the number of ways of painting these stripes such that only alternate stripes are painted by a basic colour is k. Then number of divisors of k are

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

Question Stem

Using three basic colours (Red, Blue, Green) different colour are made by mixing these two or more colours in equal proportion. The n vertical stripes are painted by all colours available, such that no two consecutive stripes have same colour.

Question

The number of ways of painting these stripes so that no two consecutive stripes have any basic color common is denoted by t_n. If $t_{2025} - K.t_{2024} = M.t_{2023}$, where K and M are integers. Find the value of $K + M$.